Středoškolské úlohy

Rovnice a nerovnice:

Rozhodněte pro která $x \in \mathbb{R}$ platí následující rovnice a nerovnice:

(1) $\frac{x-2}{2x-8} \geq 1$,

(2) $\frac{x-3}{x+5} \leq \frac{2x+3}{x-4}$,

(3) $\frac{x+4}{x-2} \geq \frac{x+1}{x+3}$,

(4) $\frac{|x|-2}{x+3} \geq \frac{x+1}{x-2}$,

(5) $\frac{|x+1|}{|x-2|-3} \geq \frac{1}{x-1}$,

(6) $||x-1|-2x| = 3$,

(7) $|x+1| + |2x-3| = |2x-1| + 4$,

(8) $||x+3|-2| \geq 1$,

(9) $|x+2| - |2x-2| \geq -8$,

(10) $|x^2 + 3|x+1| - 1| = 2$,

(11) $ax^2 + 2x + 2 = a$, $a \in \mathbb{R}$,

(12) $1 \leq |ax+1| < 2$, $a \in \mathbb{R}$,

(13) $|x|x - 3ax + 5 > 0$, $a \in \mathbb{R}$,

(14) $2^{4x+1} + 3 \cdot 2^{2x+1} - 8 = 0$

(15) $3^{1+x} + 3^{1-x} < 10$,

(16) $3\left(\frac{1}{9}\right)^{t^2} = \frac{1}{9}(27)^t$,

(17) $4^x - 3 \cdot 2^{x+1} + 8 \geq 0$,

(18) $\frac{\left(\frac{1}{3}\right)^{x^2-1}}{\left(\frac{1}{9}\right)^{|x|}} > 3$,

(19) $\left(\frac{1}{2}\right)^{x^2} < 4 \cdot 8^x$,

(20) $|2^{x+2} + a| \in < -1, 3 >$, $a \in \mathbb{R}$,

(21) $\frac{2}{\sqrt{x^2-x-6}} \leq \sqrt{\frac{4}{-x^2+3x+40}}$,

(22) $\log\left(x^2 - 3x - 4\right) \geq \log\left(-x^2 + 2x + 15\right)$,

(23) $\log_{\frac{1}{3}}\left(x^2 - 3x + 3\right) \geq 0$,

(24) $\log_2^2(x) = \log_2\left(\frac{1}{16}\right) - \log_2\left(x^5\right)$,

(25) $2\log_{\frac{1}{2}}(|x|-1) \leq \log_{\frac{1}{2}}\left(x^2 - 7x + 12\right)$,

(26) $\log_{\frac{1}{2}}\left(x^2 + 3x + 2\right) \geq -3$,

(27) $\log\left(2 - |5 - |2-x||\right) \leq 0$,

(28) $\log_2\left(x^2 + c - 3\right) \in < 1, 3)$,

(29) $\sin(2x) = \sin(x)$,

(30) $2\sin(x) + \cos(x) = 1$,

(31) $2 - \cos(2x) - 3\sin(x) < 0$,

(32) $\arccos\left(-x^2 + 4x - 1\right) > \frac{\pi}{3}$,

(33) $\cos^2(x) + 2\sin(x) < a + 2$, $a \in \mathbb{R}$,

(34) $2\sin^2(|x|-1) + 3\cos(|x|-1) = 0$,

(35) $\sin^2(x) + 2\sin(x) - \cos^2(x) < 0$,

(36) $1 - |\sin(x)| = \cos^2(x)$,

(37) $\log_{\frac{1}{2}}(1 + \sin(x)) > -1$,

(38) $\mathrm{tg}(5x-1) \leq \sqrt{3}$,

(39) $\left(\frac{\left(\frac{1}{2}\right)^{x+1}}{\left(\frac{1}{4}\right)^{x-1}}\right)^{\frac{|x+2|}{|x+1|-|x+2|}} \leq 8$,

(40) $\log_2^2 x - 3\log_4 x + \log_8 4 = \frac{1}{6}$,

(41) $\log_x 3 - 6\log_3 x \geq 1$,

(42) $|2\sin(x) - 1| - |3\sin(x) + 1| > 1$,

(43) $\mathrm{tg}^2\left(x^2 - 4x + 1\right) < 3$,

(44) $(a+1)x^2 + (a-3)x - 2a + 2 > 0$,

(45) $13\cos^2\left(\frac{x+3}{2}\right) + 8\cos\left(\frac{x+3}{2}\right) - 3\sin^2\left(\frac{x+3}{2}\right) > 0.$

Kreslení grafu:

Načrtněte graf následujících funkcí:

(46) $||x-1|^2-1|-3$,

(47) $|(\operatorname{arctg}(|x|-1)|$,

(48) $\left|(\sin\left|x+\frac{\pi}{6}\right|-\frac{1}{2}\right|$,

(49) $\left|\left(\frac{1}{2}\right)^{||x|-1|-2}-1\right|$,

(50) $\log||x|-e|$,

(51) $|2\cos(x)-1|$,

(52) $\left|\frac{x-1}{2x+1}\right|$,

(53) $|\log|x-1|-2|$,

(54) $\left|e-e^{|1-x|}\right|$,

(55) $\sin\left(|x|-\frac{\pi}{6}\right)-\frac{1}{2}$.

Výsledky:

(1) $(4,6>$,

(2) $\left(-\infty,-10-\sqrt{97}\right\rangle\cup\left(-5,-10+\sqrt{97}\right\rangle\cup(4,+\infty)$,

(3) $\left(-3,-\frac{7}{4}\right\rangle\cup(2,+\infty)$,

(4) $\left(-3,\frac{-2-\sqrt{6}}{2}\right\rangle\cup\left\langle\frac{1}{8},2\right)$,

(5) $(-\infty,-1)\cup<0,1)\cup(5,+\infty)$,

(6) $\left\{-\frac{2}{3},2\right\}$,

(7) $\{-3,5\}$,

(8) $(-\infty,-6>\cup<-4,-2>\cup<0,+\infty)$,

(9) $<-4,12>$,

(10) $\left\{\frac{3-\sqrt{33}}{2},0\right\}$,

(11) Pro $a=0:-1$; $a\neq 0:\frac{-1\pm|a-1|}{a}$,

(12) Pro $a=0:\mathbb{R}$;
$a\neq 0:\left(-\frac{1}{a}-\frac{2}{|a|},-\frac{1}{a}-\frac{1}{|a|}\right\rangle\cup\left\langle-\frac{1}{a}+\frac{1}{|a|},-\frac{1}{a}+\frac{2}{|a|}\right)$,

(13) Pro $a<\frac{\sqrt{20}}{3}:\left(\frac{-3a-\sqrt{9a^2+20}}{2},+\infty\right)$; $a\geq\frac{\sqrt{20}}{3}:$
$\left(\frac{-3a-\sqrt{9a^2+20}}{2},\frac{3a-\sqrt{9a^2-20}}{2}\right)\cup\left(\frac{3a+\sqrt{9a^2-20}}{2},+\infty\right)$,

(14) $\{0\}$,

(15) $(-1,1)$,

(16) $\left\{\frac{-3\pm\sqrt{33}}{4}\right\}$,

(17) $(-\infty,1>\cup<2,+\infty)$,

(18) $(-2,2)\setminus\{0\}$,

(19) $(-\infty,-2)\cup(-1,+\infty)$,

(20) Pro $a\geq 3:\emptyset$; $a\in<-3,3):(-\infty,\log_2(-a+3)-2\rangle$;
$a<-3:\langle\log_2(-a-3)-2,\log_2(-a+3)-2\rangle$,

(21) $\left(-5,1-\sqrt{24}\right\rangle\cup\left\langle 1+\sqrt{24},8\right)$,

(22) $\left(-3,\frac{1}{4}\left(5-\sqrt{177}\right)\right\rangle\cup\left\langle\frac{1}{4}\left(5+\sqrt{177}\right),5\right)$,

(23) $<1,2>$,

(24) $\left\{\frac{1}{2},\frac{1}{16}\right\}$,

(25) $\left\langle\frac{11}{5},3\right)\cup(4,+\infty)$,

(26) $\left\langle\frac{-3-\sqrt{33}}{2},-2\right)\cup\left(-1,\frac{-3+\sqrt{33}}{2}\right\rangle$,

(27) $(-5,-4>\cup<-2,-1)\cup(5,6>\cup<8,9)$,

(28) Pro $c<5:\left(-\sqrt{11-c},-\sqrt{5-c}\right\rangle\cup\left\langle\sqrt{5-c},\sqrt{11-c}\right)$;
$c\in<5,11):\left(-\sqrt{11-c},\sqrt{11-c}\right)$; $c\geq 11:\emptyset$,

(29) $\left\{k\pi,\frac{\pi}{3}+2k\pi,\frac{5\pi}{3}+2k\pi;\ k\in\mathbb{Z}\right\}$,

(30) $\left\{2k\pi,\pi-\arcsin\left(\frac{4}{5}\right)+2k\pi;\ k\in\mathbb{Z}\right\}$,

(31) $\bigcup_{k\in\mathbb{Z}}\left(\left(2k\pi+\frac{\pi}{6},2k\pi+\frac{5\pi}{6}\right)\setminus\left\{2k\pi+\frac{\pi}{2}\right\}\right)$,

(32) $\left\langle 0,2-\sqrt{\frac{5}{2}}\right)\cup\left(2+\sqrt{\frac{5}{2}},4\right\rangle$,

(33) Pro $a>0:\mathbb{R}$; $a\leq-4:\emptyset$; $a\in(-4,0>$:
$\bigcup_{k\in\mathbb{Z}}\left(2k\pi+(-\pi-\alpha,\alpha)\right)$, kde $\alpha=\arcsin\left(1-\sqrt{-a}\right)$,

(34) $\left\{2k\pi\pm\frac{2\pi}{3}+\operatorname{sign}(k);\ k\in\mathbb{Z}\setminus\{0\}\right\}\cup\left\{\pm\left(\frac{2\pi}{3}+1\right)\right\}$,

(35) $\bigcup_{k\in\mathbb{Z}}\left(2k\pi+\left(-\pi-\arcsin\left(\frac{-1+\sqrt{3}}{2}\right),\arcsin\left(\frac{-1+\sqrt{3}}{2}\right)\right)\right)$,

(36) $\left\{\frac{k\pi}{2};k\in\mathbb{Z}\right\}$,

(37) $\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi;\ k\in\mathbb{Z}\right\}$,

(38) $\bigcup_{k\in\mathbb{Z}}\left(\frac{k\pi}{5}+\left(\frac{2-\pi}{10},\frac{\pi+3}{15}\right\rangle\right)$,

(39) $\left(-\infty,-\frac{3}{2}\right)\cup\left\langle\frac{1+\sqrt{13}}{2},+\infty\right)$,

(40) $\left\{2,\sqrt{2}\right\}$,

(41) $\left(0,\frac{1}{\sqrt{3}}\right\rangle\cup\left(1,\sqrt[3]{3}\right\rangle$,

(42) $\bigcup_{k\in\mathbb{Z}}\left(2k\pi+\left(-\pi+\arcsin\left(\frac{1}{5}\right),-\arcsin\left(\frac{1}{5}\right)\right)\setminus\left\{-\frac{\pi}{2}\right\}\right)$,

(43) Položme $x_k^{\pm}=2\pm\sqrt{3+k\pi-\frac{\pi}{3}}$, $k\in\mathbb{N}_0$ a
$y_k^{\pm}=2\pm\sqrt{3+k\pi+\frac{\pi}{3}}$, $k\in\mathbb{N}_0\cup\{-1\}$. Pak
$x\in\left(y_{-1}^-,y_{-1}^+\right)\cup\bigcup_{k\in\mathbb{N}_0}\left(\left(y_k^-,x_k^-\right)\cup\left(x_k^+,y_k^+\right)\right)$,

(44) Pro $a<-1:\left(\frac{2-2a}{a+1},1\right)$; $a=-1:(-\infty,1)$;
$a\in\left(-1,\frac{1}{3}\right):(-\infty,1)\cup\left(\frac{2-2a}{a+1},+\infty\right)$;
$a\geq\frac{1}{3}:\left(-\infty,\frac{2-2a}{a+1}\right)\cup(1,+\infty)$,

(45) $\bigcup_{k\in\mathbb{Z}}\left(4k\pi-3+\left(-2\arccos\left(\frac{1}{4}\right),2\arccos\left(\frac{1}{4}\right)\right)\right)\cup$
$\bigcup_{k\in\mathbb{Z}}\left(4k\pi-3+\left(2\arccos\left(-\frac{3}{4}\right),4\pi-2\arccos\left(-\frac{3}{4}\right)\right)\right)$.

① $\frac{x-2}{2x-8} \overset{(*)}{\geq} 1$; a) $2x-8>0 \iff x>4$: $(*) \iff x-2 \geq 2x-8 \iff 6 \geq x$: $x \in (4,6\rangle$

b) $2x-8<0 \iff x<4$: $(*) \iff x-2 \leq 2x-8 \iff 6 \leq x$: $x \in \emptyset$

Výsledek: $x \in (4,6\rangle$

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② $\frac{x-3}{x+5} \overset{(*)}{\leq} \frac{2x+3}{x-4}$; a) $(x+5)(x-4)>0 \iff x \in (-\infty,-5)\cup(4,+\infty)$: $(*) \iff x^2-7x+12 \leq 2x^2+13x+15$

$\iff x^2+20x+3 \geq 0 \iff x \in (-\infty,x_1\rangle \cup \langle x_2,+\infty)$, $x_{1,2}=\frac{-20\pm\sqrt{400-12}}{2}=-10\pm\sqrt{97}$

$-10-\sqrt{97}<-5<-10+\sqrt{97}<0<4$: $x \in (-\infty,-10-\sqrt{97}\rangle \cup (4,+\infty)$

b) $(x+5)(x-4)<0 \iff x \in (-5,4)$: $(*) \iff x \in \langle x_1,x_2\rangle$: $x \in (-5,-10+\sqrt{97}\rangle$

Výsledek: $x \in (-\infty,-10-\sqrt{97}\rangle \cup (-5,-10+\sqrt{97}\rangle \cup (4,+\infty)$

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③ $\frac{x+4}{x-2} \overset{(*)}{\geq} \frac{x+1}{x+3}$; a) $(x-2)(x+3)>0 \iff x \notin \langle -3,2\rangle$: $(*) \iff x^2+7x+12 \geq x^2-x-2$

$\iff 8x \geq -14 \iff x \geq -\frac{7}{4}$: $x \in (2,+\infty)$

b) $(x-2)(x+3)<0 \iff x \in (-3,2)$: $(*) \iff x \leq -\frac{7}{4}$: $x \in (-3,-\frac{7}{4}\rangle$

Výsledek: $x \in (-3,-\frac{7}{4}\rangle \cup (2,+\infty)$

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④ $\frac{|x|-2}{x+3} \overset{(*)}{\geq} \frac{x+1}{x-2}$; a) $x<-3$: $(*) \iff -x^2+4 \geq x^2+4x+3 \iff 2x^2+4x-1 \leq 0 \iff x \in \langle x_1,x_2\rangle$

$x_{1,2}=\frac{-4\pm\sqrt{16+8}}{4}=-1\pm\frac{\sqrt{6}}{2}$; $-3<-1-\frac{\sqrt{6}}{2}<0<-1+\frac{\sqrt{6}}{2}<2 \implies$

$\langle x_1,x_2\rangle \cap (-\infty,-3)=\emptyset$

b) $x \in (-3,0\rangle$: $(*) \iff x \notin (x_1,x_2) \implies x \in (-3,-1-\frac{\sqrt{6}}{2}\rangle$

c) $x \in \langle 0,2)$: $(*) \iff x^2-4x+4 \leq x^2+4x+3 \iff 8x \geq 1 \iff x \geq \frac{1}{8}$: $x \in \langle \frac{1}{8},2)$

d) $x>2$: $(*) \iff x \leq \frac{1}{8}$: $x \in \emptyset$

Výsledek: $x \in (-3,-1-\frac{\sqrt{6}}{2}\rangle \cup \langle \frac{1}{8},2)$

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⑤ $\frac{|x+1|}{|x-2|-3} \overset{(*)}{\geq} \frac{1}{x-1}$; a) $x \leq -1$ ($x \neq -1$: $x<-1$): $(*) \iff \frac{-x-1}{-x-1} \geq \frac{1}{x-1} \underset{x-1<0}{\iff} x-1 \leq 1$ ✓

$x \in (-\infty,-1)$

b) $x \in (-1,1)$: $(*) \iff \frac{x+1}{-x-1} \geq \frac{1}{x-1} \underset{x-1<0}{\iff} 1-x \leq 1 \iff x \geq 0$: $x \in \langle 0,1)$

c) $x \in (1,2\rangle$: $(*) \iff x \leq 0$: $x \in \emptyset$

d) $x \geq 2$: $(*) \iff \frac{x+1}{x-5} \geq \frac{1}{x-1}$ i) $x \in \langle 2,5)$: $(*) \iff x^2-1 \leq x-5 \iff x^2-x+4 \leq 0$ nelze.

ii) $x>5$: $(*) \iff x^2-x+4 \geq 0$ vždy $\implies x \in (5,+\infty)$

Výsledek: $x \in (-\infty,-1) \cup \langle 0,1) \cup (5,+\infty)$

⑥ $||x-1|-2x| \overset{(*)}{=} 3$; a) $x \le 1$: $(*) \iff |1-3x| = 3 \iff |x-\frac{1}{3}| = 1 \iff x \in \{-\frac{2}{3}, \frac{4}{3}\}$
$\Rightarrow \underline{x = -\frac{2}{3}}$

b) $x \ge 1$: $(*) \iff |-x-1| = 3 \iff |x+1| = 3 \iff x \in \{-4, 2\} \overset{x \ge 1}{\Rightarrow} \underline{x = 2}$

Výsledek: $x \in \{-\frac{2}{3}, 2\}$

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⑦ $|x+1| + |2x-3| \overset{(*)}{=} |2x-1| + 4$; a) $x \le -1$: $(*) \iff -x-1-2x+3 = -2x+1+4 \iff \underline{x = -3} \le -1$ ✓

b) $x \in \langle -1, \frac{1}{2} \rangle$; $(*) \iff x+1-2x+3 = -2x+1+4 \iff x = 1 \notin \langle -1, \frac{1}{2} \rangle$.

c) $x \in \langle \frac{1}{2}, \frac{3}{2} \rangle$; $(*) \iff x+1-2x+3 = 2x-1+4 \iff 3x = 1 \Rightarrow x = \frac{1}{3} \notin \langle \frac{1}{2}, \frac{3}{2} \rangle$.

d) $x \ge \frac{3}{2}$; $(*) \iff x+1+2x-3 = 2x-1+4 \iff \underline{x = 5} \ge \frac{3}{2}$ ✓

Výsledek: $x \in \{-3, 5\}$

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⑧ $||x+3|-2| \ge 1 \iff |x+3| \in (-\infty, 1\rangle \cup \langle 3, +\infty) \iff \underline{x \in (-\infty, -6\rangle \cup \langle -4, -2\rangle \cup \langle 0, +\infty)}$.

Vzdálenost x od -3 je ≤ 1 nebo ≥ 3.

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⑨ $|x+2| - |2x-2| \overset{(*)}{\ge} -8$; a) $x \le -2$: $(*) \iff -x-2+2x-2 \ge -8 \iff x \ge -4 \Rightarrow \underline{x \in \langle -4, -2 \rangle}$

b) $x \in \langle -2, 1 \rangle$: $(*) \iff x+2+2x-2 \ge -8 \iff 3x \ge -8 \iff x \ge -\frac{8}{3} < -2 \Rightarrow \underline{x \in \langle -2, 1 \rangle}$

c) $x \ge 1$: $(*) \iff x+2-2x+2 \ge -8 \iff x \le 12 \Rightarrow \underline{x \in \langle 1, 12 \rangle}$

Výsledek: $x \in \langle -4, 12 \rangle$

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⑩ $|x^2 + 3|x+1| - 1| \overset{(*)}{=} 2$; a) $x \le -1$: $(*) \iff |x^2-3x-3-1| = 2 \iff x^2-3x-4 = \pm 2$

a1) $x^2-3x-4 = 2 \iff x^2-3x-6 = 0$, $x_{1,2} = \frac{3 \pm \sqrt{9+24}}{2} = \frac{3 \pm \sqrt{33}}{2}$, $\frac{3+\sqrt{33}}{2} \not\le -1$, $\underline{\frac{3-\sqrt{33}}{2} \le -1}$

a2) $x^2-3x-4 = -2 \iff x^2-3x-2 = 0$, $x_{1,2} = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2} \not\le -1$

b) $x \ge -1$: $(*) \iff |x^2+3x+3-1| = 2 \iff x^2+3x+2 = \pm 2$

b1) $x^2+3x+2 = 2 \iff x^2+3x = 0$, $x_{1,2} = \begin{cases} 0 \ge -1 \\ -3 \not\ge -1 \end{cases}$

b2) $x^2+3x+2 = -2 \iff x^2+3x+4 = 0$ ← nemá řešení

Výsledek: $x \in \{\frac{3-\sqrt{33}}{2}, 0\}$

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⑪ $ax^2+2x+2 \overset{(*)}{=} a$, $a \in \mathbb{R}$; i) $a = 0$: $(*) \iff 2x+2 = 0 \iff \underline{x = -1}$

ii) $a \ne 0$: $(*) \iff ax^2+2x+2-a = 0$; $x_{1,2} = \frac{-2 \pm \sqrt{4-4a(2-a)}}{2a} \in \mathbb{R} \Leftarrow 4-4a(2-a) = 4(a-1)^2 \ge 0$

$\Rightarrow$ ~~$x_{1,2}$~~ $\underline{x_{1,2} = \frac{-1 \pm |a-1|}{a}}$

12 $1 \le |ax+1| < 2$, $a \in \mathbb{R}$

$\Updownarrow$ Vzdálenost ax od -1 je ≥ 1 a < 2.

$ax \in (-3,-2\rangle \cup \langle 0,1) \Rightarrow$

i) $a=0$: $x \in \mathbb{R}$

ii) $a>0$: $x \in \left(-\frac{3}{a}, -\frac{2}{a}\right\rangle \cup \left\langle 0, \frac{1}{a}\right)$

iii) $a<0$: $x \in \left(\frac{1}{a}, 0\right\rangle \cup \left\langle -\frac{2}{a}, -\frac{3}{a}\right)$

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13 $|x|x - 3ax + 5 \overset{(*)}{>} 0$, $a \in \mathbb{R}$

i) $x \le 0$: $(*) \iff x^2 + 3ax - 5 < 0$

$x_{1,2} = \frac{-3a \pm \sqrt{9a^2+20}}{2}$, $9a^2+20 \ge 0$

$\sqrt{9a^2+20} \ge |3a| \Rightarrow \frac{-3a+\sqrt{9a^2+20}}{2} > 0$ ✗

$\frac{-3a-\sqrt{9a^2+20}}{2} < 0$ ✓ $\quad x \in (x_1, x_2)$

$\Rightarrow x \in \left(\frac{-3a-\sqrt{9a^2+20}}{2}, 0\right\rangle$, $a \in \mathbb{R}$

ii) $x \ge 0$: $(*) \iff x^2 - 3ax + 5 > 0$

$x_{1,2} = \frac{3a \pm \sqrt{9a^2-20}}{2}$

I) $9a^2 - 20 < 0 \iff a \in \left(-\frac{2}{3}\sqrt{5}, \frac{2}{3}\sqrt{5}\right)$: $x \in \mathbb{R} \Rightarrow x \in \langle 0, +\infty)$

II) $a \le -\frac{2}{3}\sqrt{5}$: $x \in (-\infty, x_1) \cup (x_2, +\infty)$

$\overset{a<0}{}$ $\sqrt{9a^2-20} < |3a| \Rightarrow x_{1,2} < 0 \Rightarrow x \in \langle 0, +\infty)$

III) $a \ge \frac{2}{3}\sqrt{5} > 0 \Rightarrow x_{1,2} > 0 \Rightarrow x \in \langle 0, x_1) \cup (x_2, +\infty)$

Výsledek: $a < \frac{2}{3}\sqrt{5}$: $x \in \left(\frac{-3a-\sqrt{9a^2+20}}{2}, +\infty\right)$

$a \ge \frac{2}{3}\sqrt{5}$: $x \in \left(\frac{-3a-\sqrt{9a^2+20}}{2}, \frac{3a-\sqrt{9a^2-20}}{2}\right) \cup \left(\frac{3a+\sqrt{9a^2-20}}{2}, +\infty\right)$

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14 $2^{4x+1} + 3 \cdot 2^{2x+1} - 8 \overset{(*)}{=} 0$; $t = 2^{2x}$: $(*) \iff 2t^2 + 6t - 8 = 0 \iff t^2 + 3t - 4 = 0 \Rightarrow t_{1,2} = \begin{cases} -4 \\ 1 \end{cases}$

$2^{2x} \ne -4$, $2^{2x} = 1 \Rightarrow x = 0$

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15 $3^{1+x} + 3^{1-x} \overset{(*)}{<} 10$; $t = 3^x$: $(*) \iff 3t + \frac{3}{t} < 10 \overset{t>0}{\iff} 3t^2 - 10t + 3 < 0$

$t_{1,2} = \frac{10 \pm \sqrt{100-36}}{6} = \begin{cases} 3 \\ \frac{1}{3} \end{cases} \Rightarrow t \in \left(\frac{1}{3}, 3\right) \Rightarrow x \in (-1, 1)$

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16 $3\left(\frac{1}{9}\right)^{t^2} = \frac{1}{9}(27)^t \iff \left(\frac{1}{3}\right)^{2t^2-1} = \left(\frac{1}{3}\right)^{-3t+2} \iff 2t^2 - 1 = -3t + 2 \iff 2t^2 + 3t - 3 = 0$

$\left(\frac{1}{3}\right)^y$ je prostá fce

$t_{1,2} = \frac{-3 \pm \sqrt{9+24}}{4} = \frac{-3 \pm \sqrt{33}}{4}$

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17 $4^x - 3 \cdot 2^{x+1} + 8 \overset{(*)}{\ge} 0$; $2^x = t > 0$: $(*) \iff t^2 - 6t + 8 \ge 0$; $t_{1,2} = \frac{6 \pm \sqrt{36-32}}{2} = \begin{cases} 4 \\ 2 \end{cases}$

$\Rightarrow t \in (-\infty, 2\rangle \cup \langle 4, +\infty) \Rightarrow x \in (-\infty, 1\rangle \cup \langle 2, +\infty)$

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18 $\frac{\left(\frac{1}{3}\right)^{x^2-1}}{\left(\frac{1}{9}\right)^{|x|}} > 3 \iff \left(\frac{1}{3}\right)^{x^2-1-2|x|} > \left(\frac{1}{3}\right)^{-1} \iff x^2 - 2|x| - 1 < -1 \iff x^2 - 2|x| < 0 \iff |x| \in (0, 2)$

$\left(\frac{1}{3}\right)^y$ je kles.

$x \in (-2, 0) \cup (0, 2)$

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19 $\left(\frac{1}{2}\right)^{x^2} < 4 \cdot 8^x \iff \left(\frac{1}{2}\right)^{x^2} < \left(\frac{1}{2}\right)^{-3x-2} \iff x^2 > -3x - 2 \iff x^2 + 3x + 2 > 0$, $x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \begin{cases} -1 \\ -2 \end{cases}$

$\left(\frac{1}{2}\right)^y$ je kles.

$\Rightarrow x \in (-\infty, -2) \cup (-1, +\infty)$

(20) $|2^{x+2}+a| \in \langle -1,3\rangle$, $a\in\mathbb{R}$; Vzdálenost 2^{x+2} od $-a$ je ≤ 3: $2^{x+2}\in\langle -a-3,-a+3\rangle$

i) $-a+3\le 0$: $\underline{x\in\emptyset}$ $(2^{x+2}>0)$; ii) $-a+3>0 \wedge -a-3\le 0 \Leftrightarrow a\in\langle -3,3)$: $\underline{x\in(-\infty,\log_2(-a+3)-2\rangle}$

$\Updownarrow$ $\underline{a\ge 3}$ iii) $-a-3>0 \Leftrightarrow a<-3$: $x+2\in\langle \log_2(-a-3), \log_2(-a+3)\rangle \Leftrightarrow x\in\langle \log_2(-a-3)-2, \log_2(-a+3)-2\rangle$

Výsledek: $a\ge 3$: $\emptyset$; $a\in\langle -3,3)$: $(-\infty, \log_2(-a+3)-2\rangle$; $a<-3$: $\langle \log_2(-a-3)-2, \log_2(-a+3)-2\rangle$

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(21) $\dfrac{2}{\sqrt{x^2-x-6}} \le \sqrt{\dfrac{4}{-x^2+3x+40}}$; $\overset{\mathrm{I)}}{x^2-x-6>0} \wedge \overset{\mathrm{II)}}{-x^2+3x+40>0} \wedge \dfrac{4}{x^2-x-6} \overset{\mathrm{III)}}{\le} \dfrac{4}{-x^2+3x+40}$

√ je rostoucí

I) $x^2-x-6>0$; $x_{1,2}=\dfrac{1\pm\sqrt{1+24}}{2} \begin{cases}3\\-2\end{cases} \Rightarrow \underline{x\in(-\infty,-2)\cup(3,+\infty)}$

II) $-x^2+3x+40>0$; $x_{1,2}=\dfrac{-3\pm\sqrt{9+160}}{-2} \begin{cases}8\\-5\end{cases} \Rightarrow \underline{x\in(-5,8)}$

III) $\dfrac{4}{x^2-x-6} \le \dfrac{4}{-x^2+3x+40} \overset{\mathrm{I)},\mathrm{II)}}{\Leftrightarrow} -x^2+3x+40 \le x^2-x-6 \Leftrightarrow 2x^2-4x-46\ge 0 \Leftrightarrow$

$x^2-2x-23\ge 0$; $x_{1,2}=\dfrac{2\pm\sqrt{4+92}}{2}=1\pm2\sqrt6$, $-5<1-2\sqrt6<-2<3<1+2\sqrt6<8$

$\Rightarrow x\in(-\infty,1-2\sqrt6\rangle\cup\langle 1+2\sqrt6,+\infty)$

$\Rightarrow \mathrm{I}\wedge\mathrm{II}\wedge\mathrm{III}$: $\underline{\underline{(-5,1-2\sqrt6\rangle\cup\langle 1+2\sqrt6,8)}}$

---

(22) $\log(x^2-3x-4)\ge\log(-x^2+2x+15)$; $\overset{\mathrm{I)}}{x^2-3x-4>0} \wedge \overset{\mathrm{II)}}{-x^2+2x+15>0} \wedge$ log je rostoucí

$\overset{\mathrm{III)}}{x^2-3x-4\ge -x^2+2x+15}$

I) $x^2-3x-4>0$; $x_{1,2}=\dfrac{3\pm\sqrt{9+16}}{2} \begin{cases}4\\-1\end{cases} \Rightarrow \underline{x\in(-\infty,-1)\cup(4,+\infty)}$

II) $-x^2+2x+15>0$; $x_{1,2}=\dfrac{-2\pm\sqrt{4+60}}{-2} \begin{cases}5\\-3\end{cases} \Rightarrow \underline{x\in(-3,5)}$

III $x^2-3x-4\ge -x^2+2x+15 \Leftrightarrow 2x^2-5x-19\ge 0$; $x_{1,2}=\dfrac{5\pm\sqrt{25+152}}{4}=\dfrac{5\pm\sqrt{177}}{4}$

$\Downarrow$ $\underline{x\in(-\infty,x_1\rangle\cup\langle x_2,+\infty)}$

$13<\sqrt{177}<14 \Rightarrow -3<\dfrac{5-\sqrt{177}}{4}<-1<4<\dfrac{5+\sqrt{177}}{4}<5$

$\Rightarrow$ Výsledek: $\mathrm{I}\wedge\mathrm{II}\wedge\mathrm{III}$: $\underline{\underline{\left(-3,\dfrac{5-\sqrt{177}}{4}\right\rangle\cup\left\langle\dfrac{5+\sqrt{177}}{4},5\right)}}$

---

(23) $\log_{\frac13}(x^2-3x+3)\ge 0$; $\Leftrightarrow \log_{\frac13}(x^2-3x+3)\ge\log_{\frac13}(1) \Leftrightarrow 0<x^2-3x+3\le 1 \Leftrightarrow x^2-3x+3>0 \wedge$

$\log_{\frac13}$ je klesající

vždy $\Leftarrow x_{1,2}=\dfrac{3\pm\sqrt{9-12}}{2}\notin\mathbb{R}$

$x^2-3x+2\le 0$; $x_{1,2}=\dfrac{3\pm\sqrt{9-8}}{2} \begin{cases}2\\1\end{cases} \Rightarrow \underline{x\in\langle 1,2\rangle}$

---

(24) $\log_2^2 x \overset{(*)}{=} \log_2\frac{1}{16} - \log_2(x^5)$; $(*) \Rightarrow \underline{x>0}$: $\log_2^2 x = -4-5\log_2 x$, $t:=\log_2 x \Rightarrow$

$t^2+5t+4=0 \Rightarrow t_{1,2}=\dfrac{-5\pm\sqrt{25-16}}{2} \begin{cases}-1\\-4\end{cases} \Rightarrow \underline{x\in\left\{\frac12,\frac1{16}\right\}}$

(25) $2\log_{\frac12}(|x|-1) \le \log_{\frac12}(x^2-7x+12)$; $|x|-1 \overset{I)}{>} 0 \wedge x^2-7x+12 \overset{II)}{>} 0 \wedge$ ← $\log_{\frac12}$ je klesající

$(|x|-1)^2 \ge x^2-7x+12$

I) $|x|-1>0 \iff |x|>1: x\in(-\infty,-1)\cup(1,+\infty)$

II) $x^2-7x+12>0; x_{1,2}=\frac{7\pm\sqrt{49-48}}{2} \left\langle\begin{matrix}4\\3\end{matrix}\right. \Rightarrow x\in(-\infty,3)\cup(4,+\infty)$

III) $(|x|-1)^2 \ge x^2-7x+12 \iff x^2-2|x|+1 \ge x^2-7x+12 \iff 7x-2|x|\ge 11$:

i) $x\ge 0: 5x\ge 11 \iff x\ge\frac{11}{5} \Rightarrow x\in\langle\frac{11}{5},+\infty)$

ii) $x\le 0: 9x\ge 11 \iff x\ge\frac{11}{9}$ ✗

Výsledek: $I\cap II\cap III$: $\langle\frac{11}{5},3)\cup(4,+\infty)$

(26) $\log_{\frac12}(x^2+3x+2)\ge -3$; $\iff \log_{\frac12}(x^2+3x+2)\ge\log_{\frac12}8 \iff 0<x^2+3x+2\le 8$ ($\log_{\frac12}$)

i) $x^2+3x+2>0; x_{1,2}=\frac{-3\pm\sqrt{9-8}}{2} \left\langle\begin{matrix}-1\\-2\end{matrix}\right. \Rightarrow x\in(-\infty,-2)\cup(-1,+\infty)$

ii) $x^2+3x-6\le 0; x_{1,2}=\frac{-3\pm\sqrt{9+24}}{2} = \frac{-3\pm\sqrt{33}}{2} \Rightarrow x\in\left\langle\frac{-3-\sqrt{33}}{2},\frac{-3+\sqrt{33}}{2}\right\rangle$

Výsledek: i)∩ii): $\left\langle\frac{-3-\sqrt{33}}{2},-2\right)\cup\left(-1,\frac{-3+\sqrt{33}}{2}\right\rangle$

(27) $\log(2-|5-|2-x||)\le 0$; $\iff \log(2-|5-|2-x||)\le\log 1 \iff 0<2-|5-|2-x||\le 1$
log je rostoucí
$\iff 5-|2-x|\in(-2,-1\rangle\cup\langle 1,2) \iff$

$\iff -|5-|2-x||\in(-2,-1\rangle \iff |5-|2-x||\in\langle 1,2) \iff 5-|2-x|\in(-2,-1\rangle\cup\langle1,2)$

$\iff |2-x|\in(3,4\rangle\cup\langle 6,7) \iff x-2\in(-7,-6\rangle\cup\langle-4,-3)\cup(3,4\rangle\cup\langle 6,7)$

$-|2-x|\in(-7,-6\rangle\cup\langle-4,-3) \iff |2-x| = |x-2|$

$\iff x\in(-5,-4\rangle\cup\langle-2,-1)\cup(5,6\rangle\cup\langle 8,9)$

(28) $\log_2(x^2+c-3)\in\langle 1,3)$; $\iff 1\le\log_2(x^2+c-3)<3 \iff \log_2 2\le\log_2(x^2+c-3)<\log_2 8$

$\iff 2\le x^2+c-3<8$. i) $x^2+c-3\ge 2 \iff x^2\ge 5-c$: $\mathbb{R}: c\ge 5$

log je rostoucí

$(-\infty,-\sqrt{5-c}\rangle\cup\langle\sqrt{5-c},+\infty): c<5$

ii) $x^2+c-3<8 \iff x^2<11-c$: $\emptyset: c\ge 11$

$(-\sqrt{11-c},\sqrt{11-c}): c<11$

Výsledek: i)∩ii): $c\ge 11:\emptyset$; $c\in\langle 5,11): (-\sqrt{11-c},\sqrt{11-c})$; $c<5: (-\sqrt{11-c},-\sqrt{5-c}\rangle\cup\langle\sqrt{5-c},\sqrt{11-c})$

(29) $\sin(2x)=\sin(x)$; $\iff 2\sin x\cos x=\sin x \iff \sin x=0 \vee \cos x=\frac12$

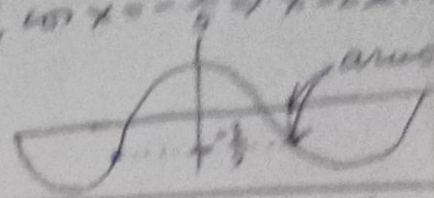
$\Leftarrow x=k\pi \qquad x=2k\pi\pm\frac{\pi}{3}$

Výsledek $x\in\{k\pi, 2k\pi\pm\frac{\pi}{3}; k\in\mathbb{Z}\}$

30 $2\sin x + \cos x = 1$; $\Leftrightarrow 2\sin x = 1-\cos x \ge 0 \Rightarrow \sin x = \sqrt{1-\cos^2 x} \Leftrightarrow$

$\Rightarrow 2\sqrt{1-\cos^2 x} = 1-\cos x \Rightarrow 4-4\cos^2 x = 1-2\cos x+\cos^2 x \Leftrightarrow 5\cos^2 x-2\cos x-3=0$

$(\cos x)_{1,2} = \frac{2\pm\sqrt{4+60}}{10} \begin{cases} 1 \\ -\frac{3}{5}\end{cases}$; $\cos x = 1 \Rightarrow x = 2k\pi, k\in\mathbb{Z}$, $\cos x = -\frac{3}{5} \Rightarrow x = 2k\pi \pm \arccos(-\frac{3}{5})$

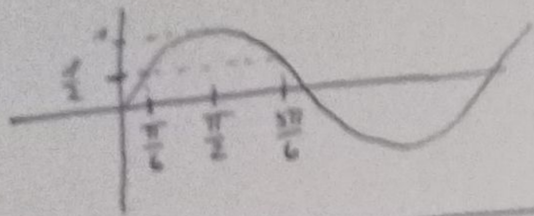


$\sin x \ge 0 \Rightarrow x \in \{2k\pi, 2k\pi + \arccos(-\frac{3}{5}); k\in\mathbb{Z}\}$

$1-\cos(2x) = \sin^2 x + \cos^2 x - (\cos^2 x - \sin^2 x) = 2\sin^2 x \Rightarrow$

31 $2-\cos(2x)-3\sin x < 0$; $(\sin x)_{1,2} = \frac{3\pm\sqrt{9-8}}{4} \begin{cases} 1 \\ \frac{1}{2}\end{cases} \Rightarrow \sin x \in (\frac{1}{2}, 1)$

$2\sin^2 x - 3\sin x + 1 < 0$:



$\Rightarrow x \in \bigcup_{k\in\mathbb{Z}} \left(2k\pi + (\frac{\pi}{6}, \frac{5\pi}{6}) \setminus \{\frac{\pi}{2}\}\right)$

32 $\arccos(-x^2+4x-1) > \frac{\pi}{3}$; $\Leftrightarrow \arccos(-x^2+4x-1) > \arccos(\frac{1}{2}) \Leftrightarrow$ arccos je klesající

$-1 \le -x^2+4x-1 < \frac{1}{2}$ i) $-x^2+4x-1 \ge -1 \Leftrightarrow x^2-4x \le 0 \Leftrightarrow x \in \langle 0,4\rangle$

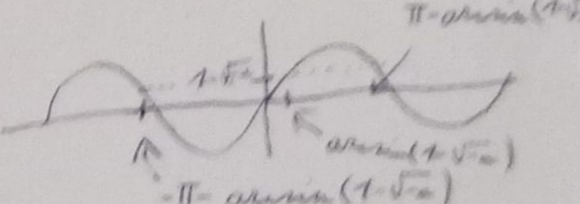
$\uparrow$ def. obor arccos ii) $-x^2+4x-1 < \frac{1}{2} \Leftrightarrow x^2-4x+\frac{3}{2} > 0$; $x_{1,2} = \frac{4\pm\sqrt{16-6}}{2} = 2\pm\sqrt{\frac{5}{2}} \Rightarrow$

$x \in (-\infty, 2-\sqrt{\frac{5}{2}}) \cup (2+\sqrt{\frac{5}{2}}, +\infty)$

Výsledek: i)∧ii): $\langle 0, 2-\sqrt{\frac{5}{2}}) \cup (2+\sqrt{\frac{5}{2}}, 4\rangle$

33 $\cos^2 x + 2\sin x \overset{(*)}{<} a+2, a\in\mathbb{R}$; $\cos^2 x + \sin^2 x = 1 \Rightarrow (*) \Leftrightarrow 1-\sin^2 x + 2\sin x < a+2$

$\Leftrightarrow \sin^2 x - 2\sin x + a + 1 > 0$; $(\sin x)_{1,2} = \frac{2\pm\sqrt{4-4-4a}}{2} = 1\pm\sqrt{-a}$.



i) $a > 0$: $\sin x \in \mathbb{R} \Rightarrow x \in \mathbb{R}$

ii) $a \le 0$: $\sin x \notin \langle 1-\sqrt{-a}, 1+\sqrt{-a}\rangle \Rightarrow \sin x < 1-\sqrt{-a}$

I) $a \le -4$: $\sin x < -1$: $x \in \emptyset$

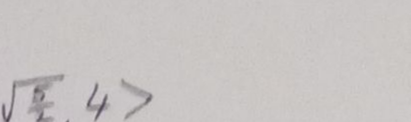
II) $a \in (-4, 0\rangle$: $x \in \bigcup_{k\in\mathbb{Z}} \left(2k\pi + (-\pi - \arcsin(1-\sqrt{-a}), \arcsin(1-\sqrt{-a}))\right)$

$|x|-1 =: t$; $\sin^2 t = 1-\cos^2 t \Rightarrow 2-2\cos^2 t + 3\cos t = 0$

$\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$

34 $2\sin^2(|x|-1) + 3\cos(|x|-1) \overset{(*)}{=} 0$;

$(\cos t)_{1,2} = \frac{-3\pm\sqrt{9+16}}{-4} \begin{cases} 2 > 1 \times \\ -\frac{1}{2} \Rightarrow \end{cases}$



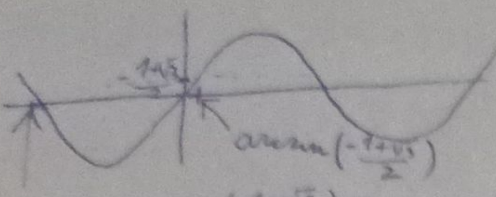
$|x|-1 = t = 2k\pi \pm \underbrace{\arccos(-\frac{1}{2})}_{\frac{2\pi}{3}}$, $k\in\mathbb{Z}$. ($|x|-1 \ge -1 \Rightarrow k > 0$ nebo $k = 0$ a znam. +)

$\Rightarrow |x| \in \{2k\pi \pm \frac{2\pi}{3} + 1, k\in\mathbb{N}\} \cup \{\frac{2\pi}{3}+1\}$

$\Rightarrow x \in \{2k\pi \pm \frac{2\pi}{3} + 1, -2k\pi \mp \frac{2\pi}{3} - 1; k\in\mathbb{N}\} \cup \{\pm(\frac{2\pi}{3}+1)\}$

(35) $\sin^2 x + 2\sin x - \cos^2 x \overset{(*)}{<} 0$; $\cos^2 x + \sin^2 x = 1 \Rightarrow (*) \Leftrightarrow 2\sin^2 x + 2\sin x - 1 < 0$

$(\sin x)_{1,2} = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-1\pm\sqrt{3}}{2}$, $\Rightarrow \sin x \in (\underbrace{\frac{-1-\sqrt3}{2}}_{<-1}, \frac{-1+\sqrt3}{2}) \Leftrightarrow \sin x < \frac{-1+\sqrt3}{2}$



$-\pi - \arcsin(\frac{-1+\sqrt3}{2}) \Leftarrow$ symetrie

$\Rightarrow x \in \bigcup_{k\in\mathbb{Z}} \left(2k\pi + \left(-\pi - \arcsin\left(\frac{-1+\sqrt3}{2}\right), \arcsin\left(\frac{-1+\sqrt3}{2}\right)\right)\right)$

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(36) $1 - |\sin(x)| \overset{(*)}{=} \cos^2 x$; $|\sin(x)| = \sqrt{\sin^2 x} = \sqrt{1-\cos^2 x} \Rightarrow (*) \Leftrightarrow 1 - \cos^2 x = \sqrt{1-\cos^2 x}$

$\Leftrightarrow \sin^2 x = |\sin x| \Leftrightarrow \sin x = 0 \vee \sin x = \pm 1 \Rightarrow x \in \{k\frac{\pi}{2}, k \in \mathbb{Z}\}$

---

(37) $\log_{\frac12}(1+\sin(x)) \overset{(*)}{>} -1$; $-1 = \log_{\frac12} 2 \Rightarrow (*) \Leftrightarrow 0 < 1 + \sin x < 2 \Leftrightarrow \sin x \neq \pm 1$

$\log_{\frac12}$ je klesající

$x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$

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(38) $\operatorname{tg}(5x-1) \overset{(*)}{\le} \sqrt3$; tg je π-per., řešme tedy nejprve pro $5x-1 \in (-\frac{\pi}{2}, \frac{\pi}{2})$:

$\sqrt3 = \operatorname{tg}(\frac{\pi}{3})$, tg je rost. na $(-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow ((*) \Leftrightarrow 5x - 1 \le \frac{\pi}{3} \Rightarrow 5x-1 \in (-\frac{\pi}{2}, \frac{\pi}{3}\rangle$

Obecně: $5x - 1 \in \bigcup_{k\in\mathbb{Z}} (k\pi + (-\frac{\pi}{2}, \frac{\pi}{3}\rangle) \Leftrightarrow x \in \bigcup_{k\in\mathbb{Z}} \left(\frac{k\pi}{5} + \left(\frac{2-\pi}{10}, \frac{\pi+3}{15}\right\rangle\right)$

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(39) $\left(\frac{(\frac12)^{x+1}}{(\frac14)^{x-1}}\right)^{\frac{|x+2|}{|x+1|-|x+2|}} \overset{(*)}{\le} 8$; $8 = (\frac12)^{-3}$, $(\frac12)^y$ je klesající $\Rightarrow ((*) \Leftrightarrow \left(\frac{(\frac12)^{x+1}}{(\frac14)^{x-1}}\right)^{\frac{|x+2|}{|x+1|-|x+2|}} \le (\frac12)^{-3}$

$= \left(\frac{(\frac12)^{x+1}}{(\frac12)^{2x-2}}\right)^{\frac{|x+2|}{|x+1|-|x+2|}} = \left(\frac12\right)^{\frac{(3-x)|x+2|}{|x+1|-|x+2|}}$

$\Leftrightarrow \frac{(3-x)|x+2|}{|x+1|-|x+2|} \overset{(+)}{\ge} -3$

a) $x \le -2$: $\Rightarrow |x+1| - |x+2| = 1 > 0 \Rightarrow (x-3)(x+2) \ge -3 \Leftrightarrow x^2 - x - 3 \ge 0$, $x_{1,2} = \frac{1\pm\sqrt{1+12}}{2} = \frac{1\pm\sqrt{13}}{2}$

$-2 < \frac{1\pm\sqrt{13}}{2} \Rightarrow x \in (-\infty, -2\rangle$

b) $x \in \langle -2, -1\rangle$: $\Rightarrow |x+1| - |x+2| = -2x - 3 > 0 \Leftrightarrow x < -\frac32$.

b1) $x \in \langle -2, -\frac32)$: $(+) \Leftrightarrow (3-x)(x+2) \ge (-3)(-2x-3) \Leftrightarrow -x^2 + x + 6 \ge 6x + 9$

$\Leftrightarrow x^2 + 5x + 3 \le 0$; $x_{1,2} = \frac{-5 \pm\sqrt{25-12}}{2} = \frac{-5\pm\sqrt{13}}{2}$; $\frac{-5-\sqrt{13}}{2} < -2 < -\frac32 < -1 < \frac{-5+\sqrt{13}}{2}$

$\Rightarrow x \in \langle -2, -\frac32)$

b2) $x \in (-\frac32, -1\rangle$: $x^2 + 5x + 3 \ge 0$; $x \in \emptyset$

c) $x \ge -1$: $|x+1| - |x+2| = -1 \Rightarrow (+) \Leftrightarrow (3-x)(x+2) \le 3 \Leftrightarrow -x^2 + x + 6 \le 3 \Leftrightarrow$

$x^2 - x - 3 \ge 0$, $x_{1,2} = \frac{1\pm\sqrt{1+12}}{2} = \frac{1\pm\sqrt{13}}{2}$ (viz a), $\frac{1-\sqrt{13}}{2} < -1 < \frac{1+\sqrt{13}}{2} \Rightarrow$

$x \in \langle \frac{1+\sqrt{13}}{2}, +\infty)$

Výsledek: $x \in (-\infty, -\frac32) \cup \langle \frac{1+\sqrt{13}}{2}, +\infty)$

(40) $\log_2^2 x - 3\log_4 x + \log_8 4 = \frac{1}{6}$; $t := \log_2 x \Rightarrow \log_4 x = \frac{t}{2}$; $\log_8 4 = \frac{2}{3} \Rightarrow$

$\Updownarrow$

$t^2 - \frac{3}{2}t + \frac{2}{3} = \frac{1}{6} \Leftrightarrow t^2 - \frac{3}{2}t + \frac{1}{2} = 0$; $t_{1,2} = \frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} - 2}}{2} \begin{cases} 1 \\ \frac{1}{2} \end{cases} \Rightarrow x \in \{2, \sqrt{2}\}$

---

(41) $\log_x 3 - 6\log_3 x \overset{(*)}{\geq} 1$; $\log_b x = \frac{\log_a x}{\log_a b}$, $x > 0$, $a, b \in (0,1) \cup (1,+\infty)$. $t := \log_x 3 \Rightarrow$

$\log_3 x = \frac{\log_x x}{\log_x 3} = \frac{1}{t}$, $x \in (0,1) \cup (1,+\infty) \Rightarrow (*) \Leftrightarrow t - \frac{6}{t} \geq 1 \ (+)$

a) $x < 1 \Rightarrow t < 0 \Rightarrow (+) \Leftrightarrow t^2 - t - 6 \leq 0$; $t_{1,2} = \frac{1 \pm \sqrt{1+24}}{2} \begin{cases} 3 > 0 \ \times \\ -2 \ ✓ \end{cases} \Rightarrow t \in \langle -2, 0) \Rightarrow x \in (0, \frac{1}{\sqrt{3}}\rangle$

b) $x > 1 \Rightarrow t > 0 \Rightarrow (+) \Leftrightarrow t^2 - t - 6 \geq 0$ — " — $\Rightarrow t \in \langle 3, +\infty) \Rightarrow x \in (1, \sqrt[3]{3}\rangle$

Výsledek: $x \in (0, \frac{1}{\sqrt{3}}\rangle \cup (1, \sqrt[3]{3}\rangle$

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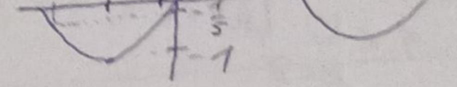
(42) $|2\sin(x) - 1| - |3\sin(x) + 1| \overset{(*)}{>} 1$; $\sin x =: t \in \langle -1, 1\rangle$.

$|2t - 1| - |3t + 1| > 1 \ (+)$

a) $t \in \langle -1, -\frac{1}{3}\rangle$: $(+) \Leftrightarrow -2t + 1 + 3t + 1 > 1 \Leftrightarrow t > -1 \Rightarrow t \in (-1, -\frac{1}{3}\rangle$

b) $t \in \langle -\frac{1}{3}, \frac{1}{2}\rangle$: $(+) \Leftrightarrow -2t + 1 - 3t - 1 > 1 \Leftrightarrow 5t < -1 \Leftrightarrow t < -\frac{1}{5} \Rightarrow t \in \langle -\frac{1}{3}, -\frac{1}{5})$

c) $t \in \langle \frac{1}{2}, 1\rangle$: $(+) \Leftrightarrow 2t - 1 - 3t - 1 > 1 \Leftrightarrow t < -3 \Rightarrow t \in \emptyset$



Dohromady: $\sin x = t \in (-1, -\frac{1}{5})$

$\alpha = \arcsin(-\frac{1}{5}) = -\arcsin(\frac{1}{5})$

$\beta = -\pi + \arcsin(\frac{1}{5}) \Rightarrow$ Výsledek: $x \in \bigcup_{k \in \mathbb{Z}} \left(2k\pi + (-\pi + \arcsin(\tfrac{1}{5}), -\arcsin(\tfrac{1}{5}))\right) \setminus \{-\frac{\pi}{2}\}$

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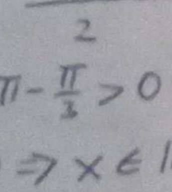
(43) $\operatorname{tg}^2(x^2 - 4x + 1) \overset{(*)}{<} 3$; $t := x^2 - 4x + 1 \Rightarrow (*) \Leftrightarrow \operatorname{tg}^2 t < 3 \Leftrightarrow \operatorname{tg} t \in (-\sqrt{3}, \sqrt{3}) \Rightarrow$

$t \in \bigcup_{k \in \mathbb{Z}} \left(k\pi + (-\frac{\pi}{3}, \frac{\pi}{3})\right)$



$k\pi - \frac{\pi}{3} \overset{(+)}{<} x^2 - 4x + 1 \overset{(\Delta)}{<} k\pi + \frac{\pi}{3}$

$(+) \Leftrightarrow x^2 - 4x + 1 + \frac{\pi}{3} - k\pi > 0$

$\tilde{x}^k_{1,2} = \frac{4 \pm \sqrt{16 - 4 - \frac{4}{3}\pi + 4k\pi}}{2} = 2 \pm \sqrt{3 + k\pi - \frac{\pi}{3}}$

$3 + k\pi - \frac{\pi}{3} > 0 \Leftrightarrow k \in \mathbb{N}_0$

$k < 0 \Rightarrow x \in \mathbb{R}$; $k \in \mathbb{N}_0 \Rightarrow x \in (-\infty, \tilde{x}^k_1) \cup (\tilde{x}^k_2, +\infty)$

$(\Delta) \Leftrightarrow x^2 - 4x + 1 - \frac{\pi}{3} - k\pi < 0$

$\hat{x}^k_{1,2} = \frac{4 \pm \sqrt{16 - 4 + \frac{4}{3}\pi + 4k\pi}}{2} = 2 \pm \sqrt{3 + k\pi + \frac{\pi}{3}}$

$3 + k\pi + \frac{\pi}{3} > 0 \Leftrightarrow k \geq -1$

$k < -1 \Rightarrow x \in \emptyset$; $k \geq -1 \Rightarrow x \in (\hat{x}^k_1, \hat{x}^k_2)$

Výsledek:

$x \in \bigcup_{k \in \mathbb{N}_0} \left((\hat{x}^k_1, \tilde{x}^k_1) \cup (\tilde{x}^k_2, \hat{x}^k_2)\right) \cup (\hat{x}^{-1}_1, \hat{x}^{-1}_2)$

(44) $(a+1)x^2+(a-3)x-2a+2>0\ (*)$ $a\neq-1$: $x_{1,2}=\dfrac{3-a\pm\sqrt{a^2-6a+9-4(a+1)(-2a+2)}}{2a+2}=$

$=\dfrac{3-a\pm\sqrt{9a^2-6a+1}}{2a+2}=\dfrac{3-a\pm|3a-1|}{2a+2}$

I) $\underline{a<-1}$: $a+1<0\Rightarrow(*)\Leftrightarrow x\in(x_1,x_2)=\left(\frac{2-2a}{a+1},1\right)$

$\Downarrow$ $x_{1,2}=\dfrac{3-a\pm(3a-1)}{2a+2}=\begin{cases}\frac{2-2a}{a+1}\\ 1\end{cases}$

II) $\underline{a=-1}$: $(*)\Leftrightarrow -4x+4>0\Leftrightarrow x\in(-\infty,1)$

III) $\underline{a\in(-1,\frac{1}{3})}$: $a+1>0\Rightarrow(*)\Leftrightarrow x\notin\langle x_1,x_2\rangle$,

$\Downarrow$ $\frac{2-2a}{a+1}>1\Rightarrow$ $\Downarrow$ $x\in(-\infty,1)\cup\left(\frac{2-2a}{a+1},+\infty\right)$

IV) $\underline{a\geq\frac{1}{3}}$: Podobně jako ↑ $\Rightarrow x\in\left(-\infty,\frac{2-2a}{a+1}\right)\cup(1,+\infty)$

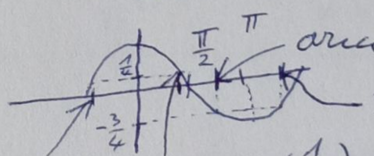
$\Downarrow$ $\frac{2-2a}{a+1}\leq1$

---

(45) $13\cos^2\left(\frac{x+3}{2}\right)+8\cos\left(\frac{x+3}{2}\right)-3\sin^2\left(\frac{x+3}{2}\right)\overset{(*)}{>}0$; $\frac{x+3}{2}=:t$, $\sin^2t=1-\cos^2t\Rightarrow$

$(*)\Leftrightarrow16\cos^2t+8\cos t-3>0$; $(\cos t)_{1,2}=\dfrac{-8\pm\sqrt{64+12\cdot16}}{32}\begin{cases}\frac{1}{4}\\ -\frac{3}{4}\end{cases}\Rightarrow$

$\cos t>\frac{1}{4}\vee\cos t<-\frac{3}{4}$



$\arccos(-\frac{3}{4})$

$2\pi-\arccos(-\frac{3}{4})$

↑ ze symetrie.

$t\in\underbrace{\bigcup_{k\in\mathbb{Z}}\left(2k\pi+\left(\left(-\arccos(\tfrac{1}{4}),\arccos(\tfrac{1}{4})\right)\cup\left(\arccos(-\tfrac{3}{4}),2\pi-\arccos(-\tfrac{3}{4})\right)\right)\right)}_{M:=}$ $\Leftarrow$ $-\arccos(\frac{1}{4})\Leftarrow\arccos(\frac{1}{4})$

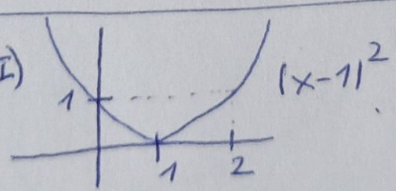
$\Rightarrow\frac{x+3}{2}\in M\Leftrightarrow x\in2M-3\Rightarrow$ Výsledek je:

$x\in\bigcup_{k\in\mathbb{Z}}\left(\left(4k\pi-3-2\arccos(\tfrac{1}{4}),4k\pi-3+2\arccos(\tfrac{1}{4})\right)\cup\left(4k\pi-3+2\arccos(-\tfrac{3}{4}),(4k+4)\pi-3-2\arccos(-\tfrac{3}{4})\right)\right)$

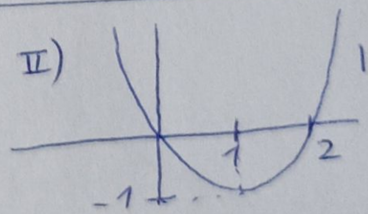
---

(46) $f(x)=||x-1|^2-1|-3$:

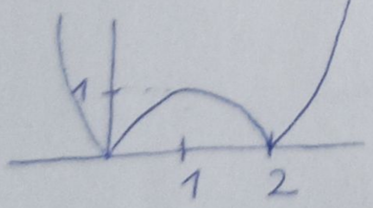
I)



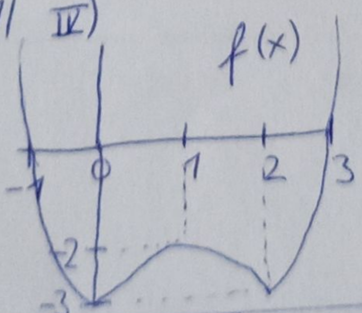
II)



III)



IV)



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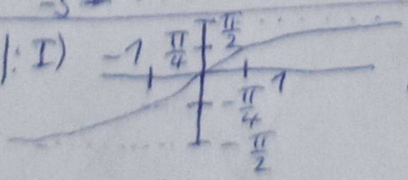
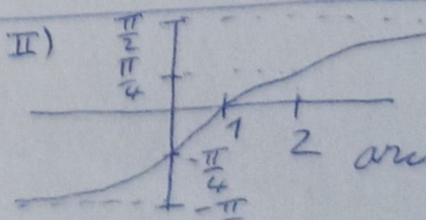
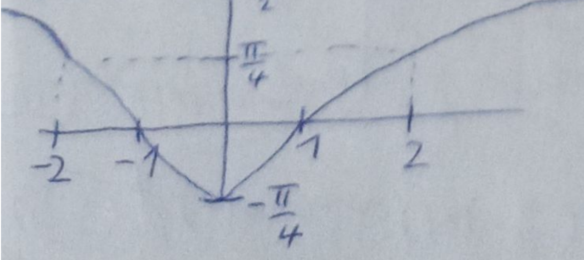
(47) $f(x)=|\operatorname{arctg}(\underbrace{|x|-1}_{z})|$: I)



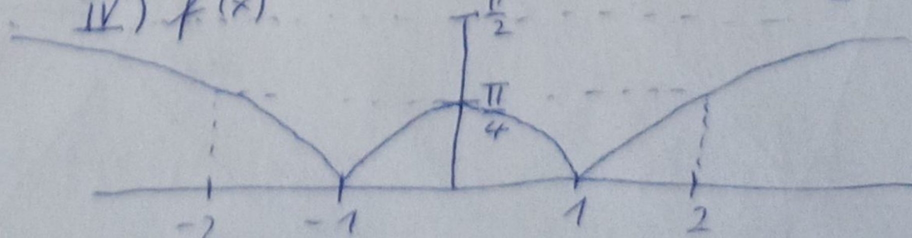
$z=y-1$ II)



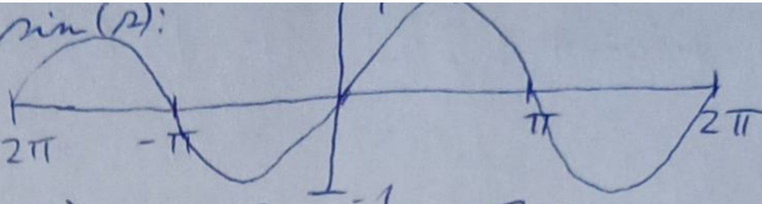
$y=|x|$ III) $\operatorname{arctg}(|x|-1)$



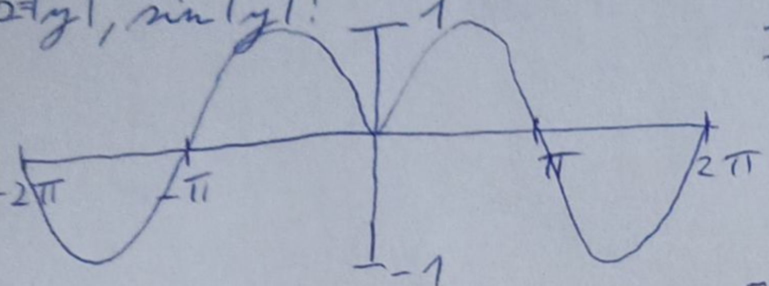
IV) $f(x)$



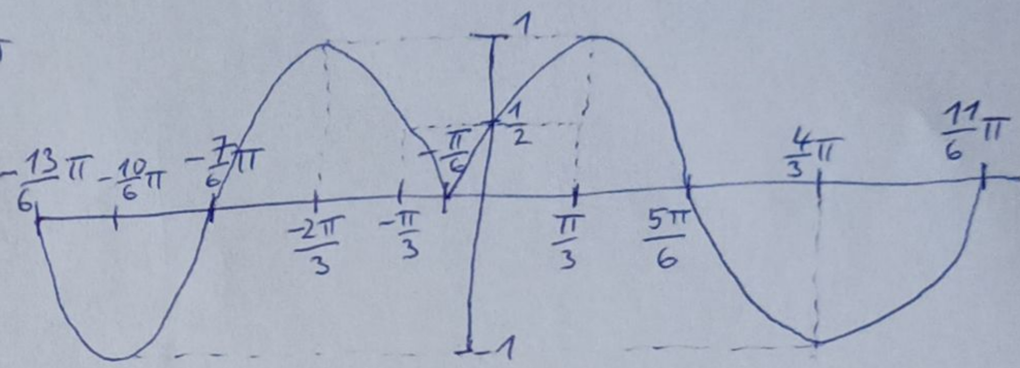
48) $f(x)=\left|\sin\left|\underbrace{\underbrace{x+\frac{\pi}{6}}_{y}}_{z}\right|-\frac{1}{2}\right|$: I) $\sin(z)$:



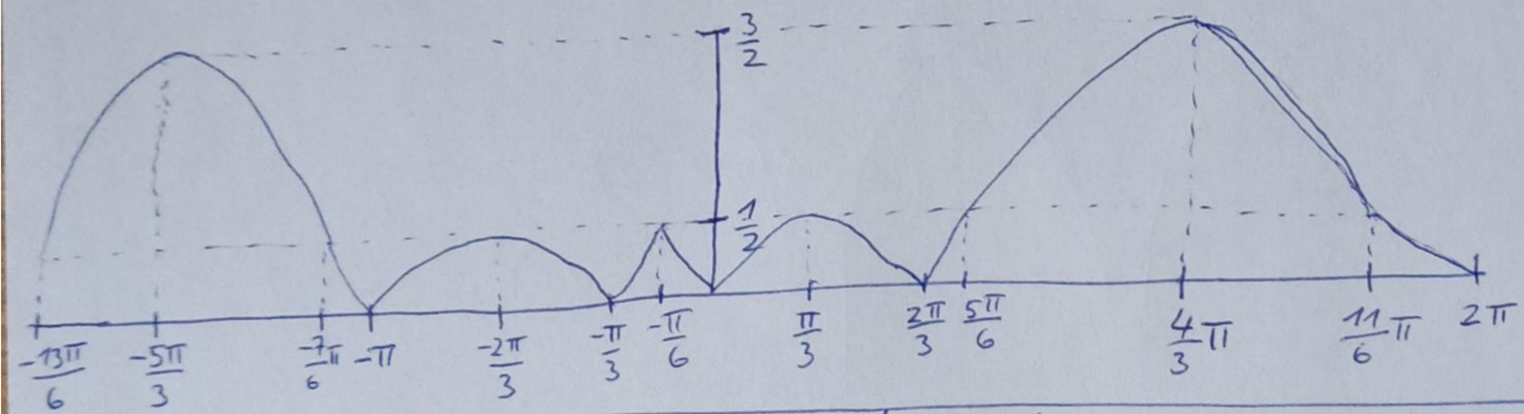
II) $z=|y|$, $\sin|y|$:



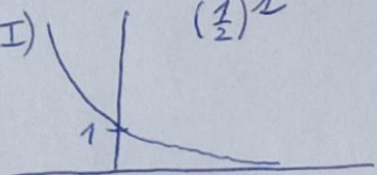
III) $y=x+\frac{\pi}{6}$ $\sin\left|x+\frac{\pi}{6}\right|$:



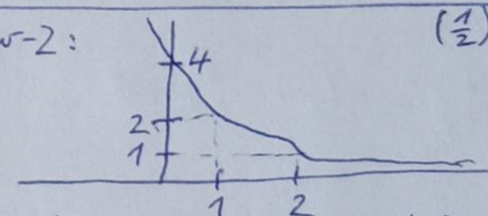
IV) $f(x)$:



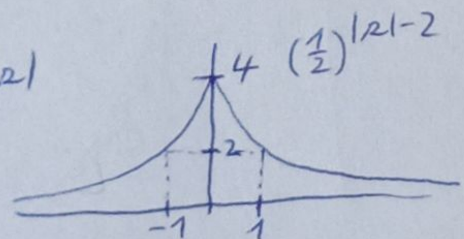
49) $f(x)=\left|\left(\frac{1}{2}\right)^{||x|-1|-2}-1\right|$: I)



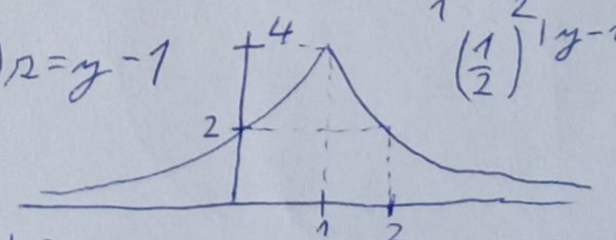
II) $z=w-2$:



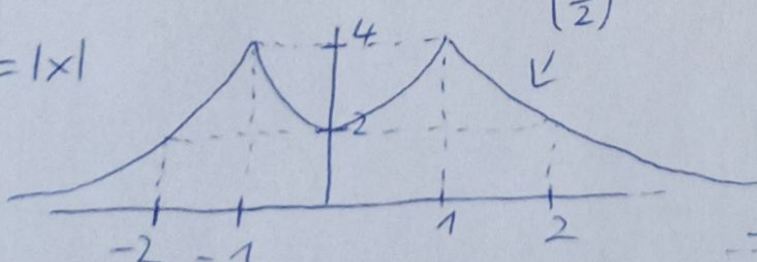
III) $w=|z|$



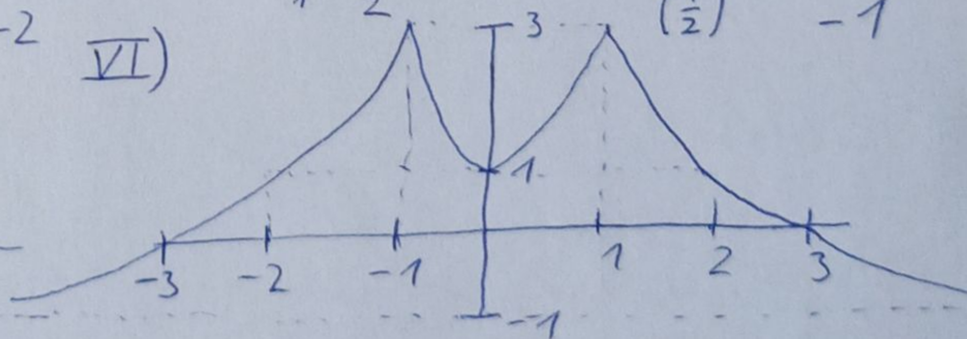
IV) $z=y-1$



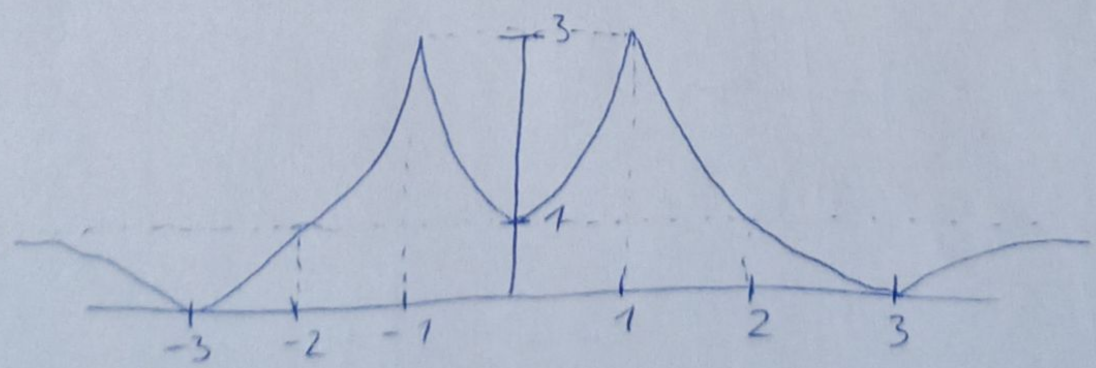
V) $y=|x|$



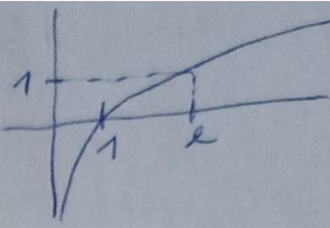
VI)



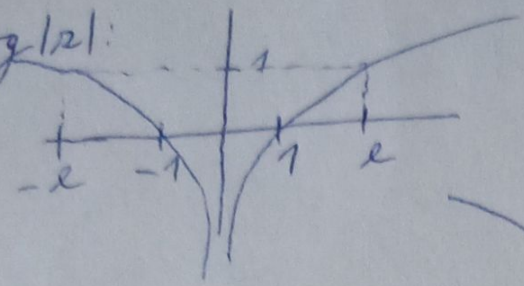
VII) $f(x)$:

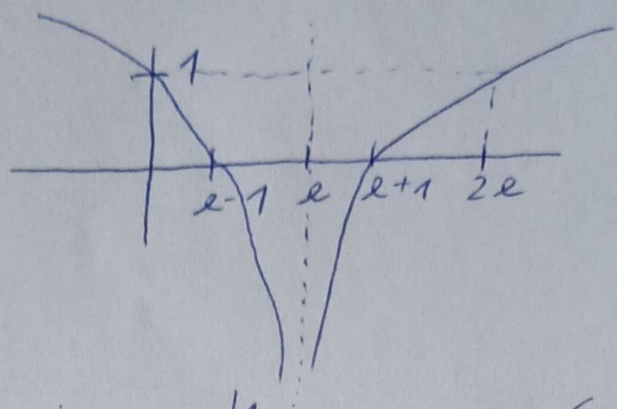


(50) $f(x)= \cancel{|x|-e}\ \log||x|-e|$: I) $\log s$



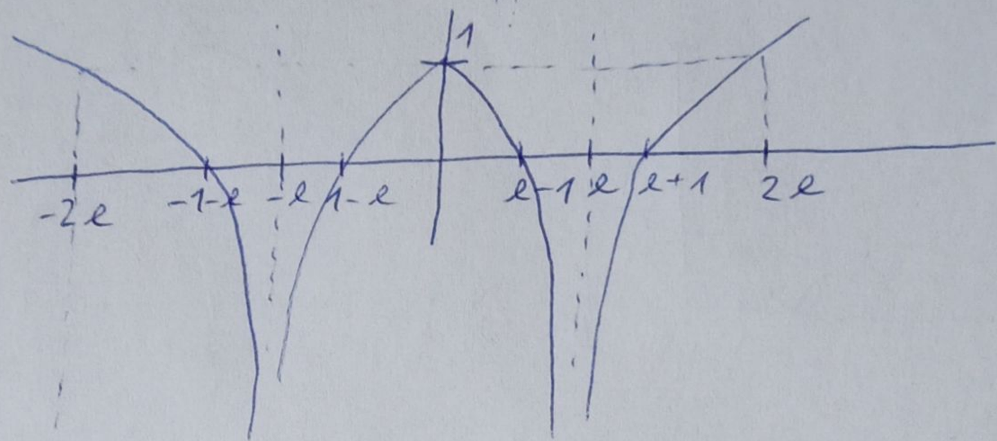
II) $s=|s|$: $\log|s|$:





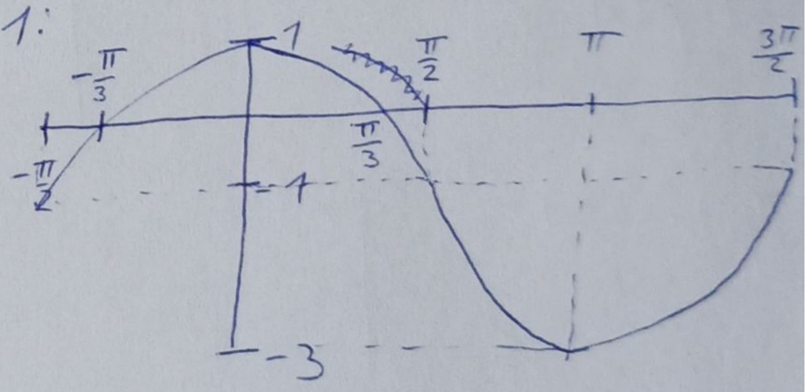
III) $s=y-e$: $\log|y-e|$:

IV) $y=|x|$: $f(x)$:

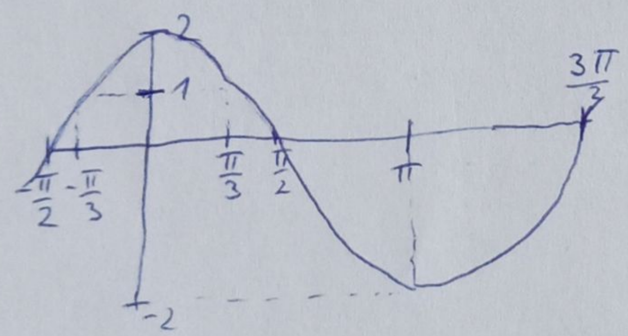


(51) $f(x)=|2\cos(x)-1|$

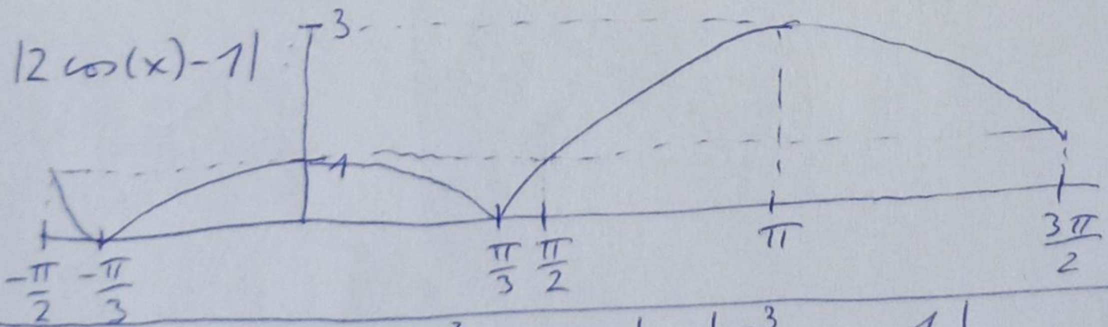
II) $2\cos x-1$:



I) $2\cos(x)$:

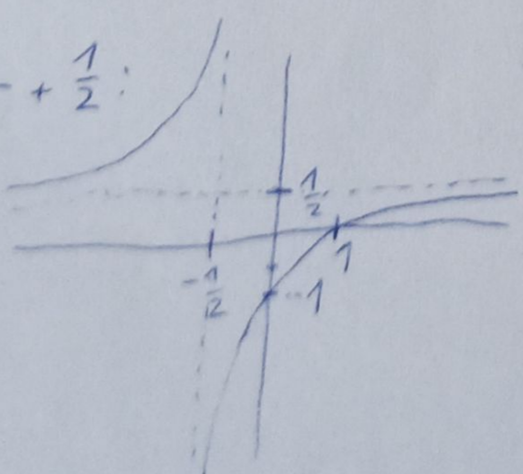


III $|2\cos(x)-1|$

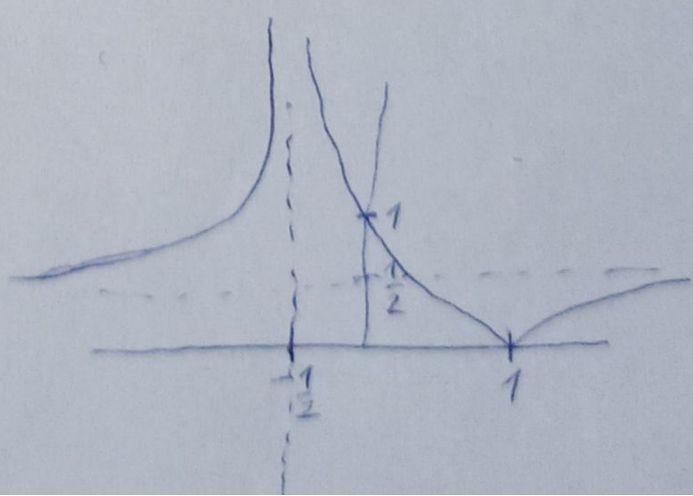


(52) $f(x)=\left|\frac{x-1}{2x+1}\right|=\left|\frac{-\frac{3}{2}}{2x+1}+\frac{1}{2}\right|=\left|\frac{-\frac{3}{4}}{x+\frac{1}{2}}+\frac{1}{2}\right|$

I) $\frac{-\frac{3}{4}}{x+\frac{1}{2}}+\frac{1}{2}$:

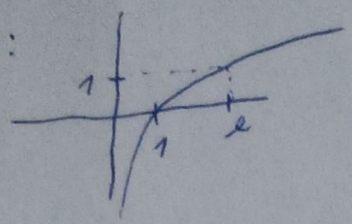


II) $f(x)$:

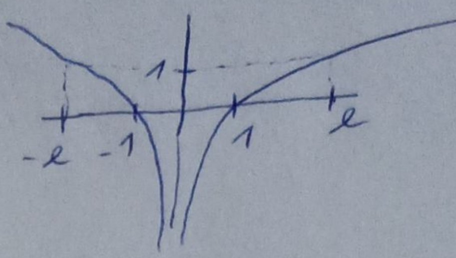


(53) $f(x) = |\log|x-1| - 2|$:

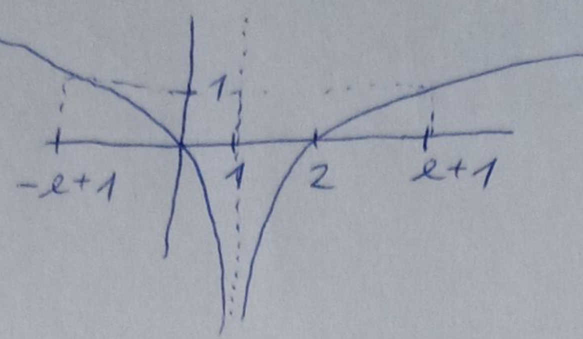
I) $\log z$:



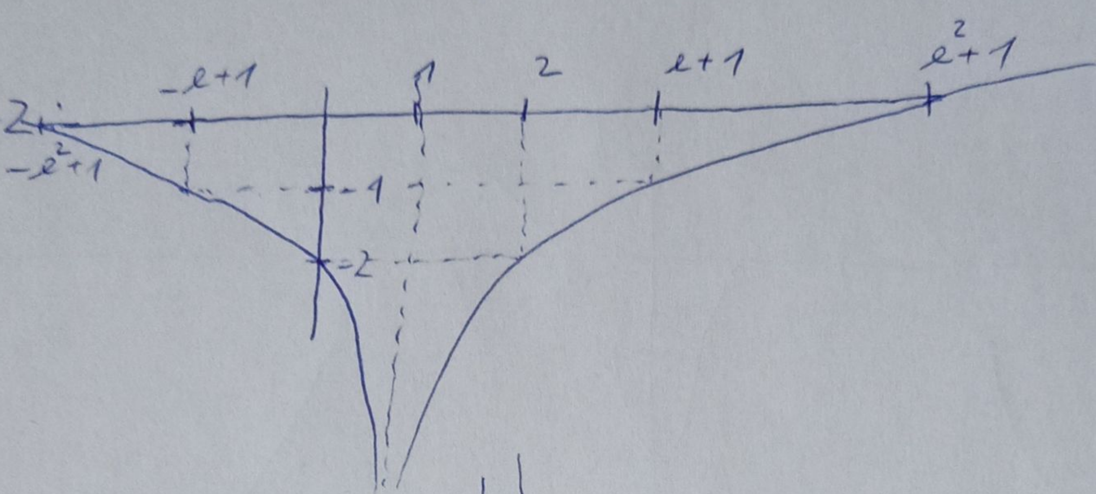
II) $z = |y|$: $\log|y|$:



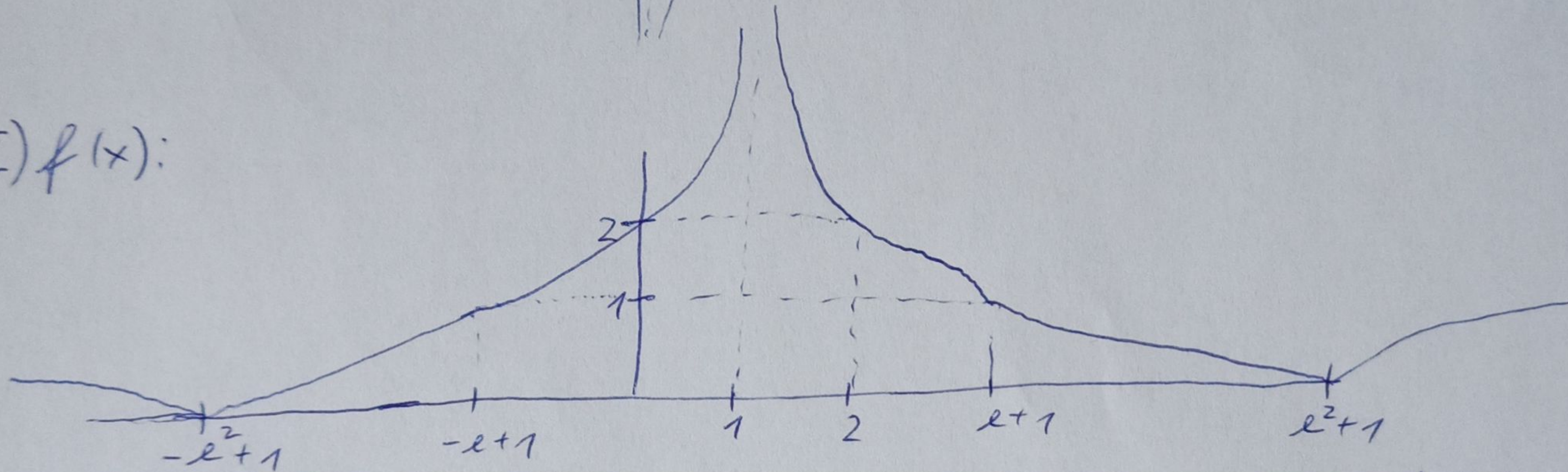
III) $y = x-1$: $\log|x-1|$:



IV) $\log|x-1| - 2$:



V) $f(x)$:



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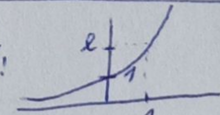
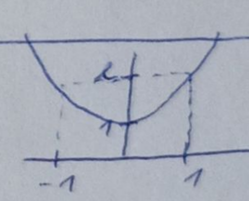
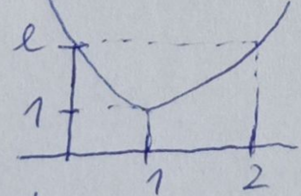
(54) $f(x) = |e - e^{|1-x|}| = |e^{|x-1|} - e|$: I) e^z:



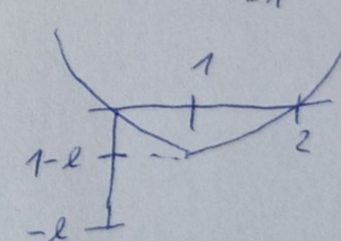
II) $z = |y|$: $e^{|y|}$:



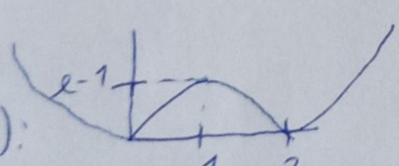
III) $y = x-1$: $e^{|x-1|}$:



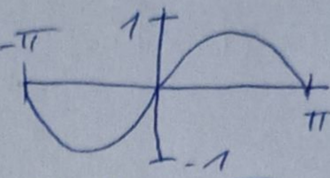
IV) $e^{|x-1|} - e$:



V) $f(x)$:

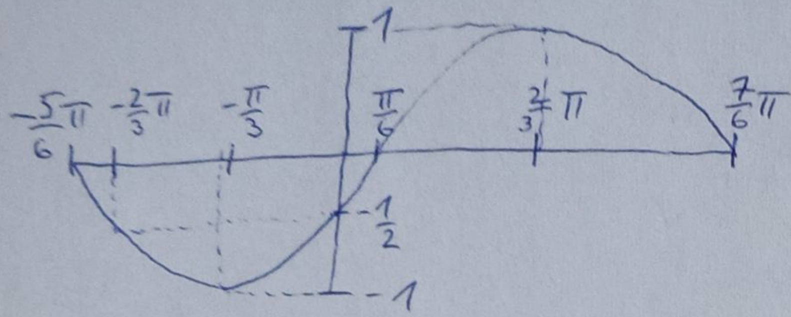


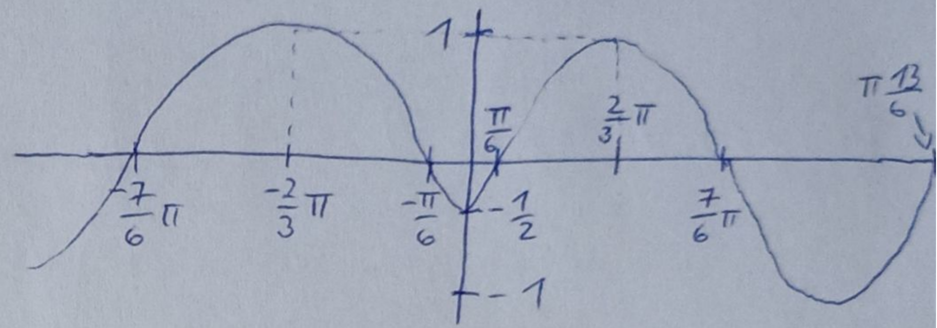
55 $f(x) = \sin\left(|x| - \frac{\pi}{6}\right) - \frac{1}{2}$: I) $\sin(r)$:



II) $r = y - \frac{\pi}{6}$: $\sin\left(y - \frac{\pi}{6}\right)$

III) $y = |x|$: $\sin\left(|x| - \frac{\pi}{6}\right)$





IV) $f(x)$: